QP CODE: 26017608

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Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE / MERCY CHANCE EXAMINATIONS, MARCH 2026

Sixth Semester

CORE COURSE - MM6CRT04 - LINEAR ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

89C6D4EB

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Write 3x3 matrix whose entries are given by $x_{ij} = (-1)^{i-j}$

2. a)Define linearly dependent rows.

   b)*Prove that in the matrix* $A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ *the columns are linearly dependent.*

3. Define a vector space and give an example of a vector space

4. *Define span S of a vector space V and Prove that S = {(1,0), (0,1)} is a spanning set of* R^2

5. Define a basis of a vector space V and write the canonical bases of R^n.

6. Define differentiation map $D : \mathbb{R}_n[X] \to \mathbb{R}_n[X]$. Is D linear?

7. If V and W are vector spaces of the same dimension n over F, then prove that V and W are isomorphic.

8. Determine the transition matrix from the ordered basis $\{(1,-1,1),(1,-2,2),(1,-2,1)\}$ of $\mathbb{R}^3$ to the natural ordered basis of $\mathbb{R}^3$.

9. Define an equivalence relation. State whether the relation of being similar is an equivalence relation on the set of $n \times n$ matrices.

10. Define characteristic polynomial and characteristic equation of a matrix.

11. Define eigen value of a linear map and the eigen vector associated with it.

12. Define diagonalizable linear map and diagonalizable matrix.

(10×2=20)

Part B

*Answer any **six** questions*

14. Show that the matrix A = $\begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$ is of rank 2 and find matrices P,Q such that PAQ = $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

15. Determine for which values of α if the matrix A = $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & \alpha \end{bmatrix}$ is invertible and find A^{-1}..

16. *Prove that the set of lower triangular nxn matrices is a subspace of the vector space*

17. Find $Im\ f$ and $Ker\ f$ when $f : \mathbb{R}^3 \to \mathbb{R}^3$ is given by $f(a, b, c) = (a + b,\ b + c,\ a + c)$.

18. a) Define rank and nullity of a linear mapping and define Dimension Theorem for finite-dimensional vector spaces over a field F in terms of rank and nullity.
    b) Find the rank and nullity of $pr_1 : \mathbb{R}^3 \to \mathbb{R}$ defined by $pr_1(x, y, z) = x$.

19. a) Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.
    b) Suppose that f is nilpotent of index p. If $x \in V$ is such that $f^{p-1}(x) \neq 0$, prove that $\{x, f(x), f^2(x), \ldots, f^{p-1}(x)\}$ is linearly independent.

20. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

21. For the nXn tridiagonal matrix An = $\begin{bmatrix} 2 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 2 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 2 & 1 & \ldots & 0 & 0 \\ . & . & . & . & ..... & . & . \\ 0 & 0 & 0 & 0 & \ldots & 2 & 1 \\ 0 & 0 & 0 & 0 & \ldots & 1 & 2 \end{bmatrix}$ Prove that det An = n + 1.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Reduce the following matrix to Hermite form $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 4 & 4 & 8 & 4 \\ 3 & 6 & 5 & 7 & 7 \end{bmatrix}$

    b) Prove that by using elementary row operation, a non-zero matrix can be transformed to a row-echelon matrix.

    c) Prove that every non-zero matrix A can be transformed to a Hermite matrix by using elementary row operations

23. a)Define dimension of a vector space V and Find the dimension of M_{mxn} R.

    b)Prove that the set W of complex matrices of the form $\begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & -\alpha \end{bmatrix}$ is a real vector space of dimension 4.

    c)Let V be a finite dimensional vector space. If W is a subspace of V . Prove that W is of finite dimension and dim W ≤ dim V. If dim W = dim V iff W = V

24. State and prove Dimension Theorem for finite-dimensional vector spaces over a field F.

25. A linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ is such that $f(1, 0, 0) = (0, 0, 1)$, $f(1, 1, 0) = (0, 1, 1)$, $f(1, 1, 1) = (1, 1, 1)$. Determine $f(x, y, z)$ for all $(x, y, z) \in \mathbb{R}^3$ and compute the matrix of f relative to the ordered basis $B = \{(1, 2, 0), (2, 1, 0), (0, 2, 1)\}$. If $g : \mathbb{R}^3 \to \mathbb{R}^3$ is the linear mapping given by $g(x, y, z) = (2x,\ y + z,\ -x)$, compute the matrix $f \circ g \circ f$ relative to the ordered basis B.

(2×15=30)